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AN APPROXIMATION METHOD FOR BOUND STATES IN QUANTUM CHROMODYNAMICS

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A method for extracting hadron states in confining field theories (QCD) from the asymptotic behavior of two-point functions and the imposition of confinement through the removal of cuts is reviewed and generalized to take explicit account of the spin-$\frac{1}{2}$ nature of the constituents. The role of perturbative corrections is discussed and explored numerically with anomalous dimension for arbitrary spin J and to sixth order for the specific example of spin-1 hadronic states. We conclude that the effects of these are significant though incapable of leading to linear Regge trajectories, and that the main features of the hadron spectrum must come from the non-perturbative inverse power corrections.

1. Introduction

QCD in SU(N) of color is believed to be a confining theory, i.e. there are no cuts corresponding to decay of hadrons into free quarks; moreover, in the $N \to \infty$ limit physical cuts from the decay of hadrons disappear and the hadronic spectrum is discrete [1]. Two-point functions of the type

$$\tilde{G}(t) = \frac{1}{(2\pi)^4} \int d^4x \; e^{ikx} \langle 0 | \mathrm{T}(O^{(1)}(x) O^{(2)}(0)) | 0 \rangle \,, \tag{1.1}$$

where t is a variable, related to k^2, that will be specified below, and where $O^{(1)}$, $O^{(2)}$ are local operators characterized by a definite Lorentz spin, will have a point spectrum, with the poles of $\tilde{G}(t)$ located at values of t determined by the masses of hadrons that can be created by the operators $O^{(1)}$, $O^{(2)}$ acting on the physical vacuum.

A scheme has been proposed [2–7] that uses an asymptotic approximation to $\tilde{G}(t)$, $G_A(t)$, such as would be generated by perturbation theory (because of asymptotic freedom) or by non-perturbative inverse power corrections [8], to construct a cutless $G(t)$ that approximates $\tilde{G}(t)$ for both asymptotic and non-asymptotic values of t. The positions and coupling strengths of the bound-state poles are thereby determined.

As we explain below, in lowest order of perturbation theory the construction describes a free particle in an infinite well. However, the details of the construction are not uniquely specified. This is because the input approximation contains too little information to specify the spin of the particle that appears in the "vacuum polarization" induced by the "currents" $O^{(1)}$, $O^{(2)}$. Better approximations (including non-perturbative inverse power effects) presumably supply that information. It turns out, however, that the limitation in the original form of the construction [4], which is appropriate for spinless particles, can be remedied at the lowest order level in a simple and physically reasonable way. For a spin-$\frac{1}{2}$ particle we are led to a new construction that emphasizes analyticity in its energy E and brings in half-integer values of angular momentum. Not only does the new algorithm give lowest order results consistent with a spin-$\frac{1}{2}$ particle in the naive bag*, but it retains the basic framework for the systematic study of the effects on the spectrum of interaction corrections to the asymptotic behavior. These corrections can be either perturbative, which give logarithmic corrections to the lowest order approximation, or inverse power corrections [8] that are non-perturbative and could be generated by instanton effects, for example**.

The general form of the asymptotic approximation will be of the form

$$G_{\mathrm{A}}(t) \underset{t\to\infty}{\longrightarrow} t^{\nu}(1+\text{non-leading terms})\,. \tag{1.2}$$

$G_{\mathrm{A}}(t)$ clearly has cuts. The confining algorithm is based on the construction [4] of a cutless $G(t)$ that is asymptotically closer to $G_{\mathrm{A}}(t)$ than any inverse power of t, and therefore approximates $\tilde{G}(t)$ everywhere. The tool is a set of moment conditions that have a unique solution once CDD singularities [6] are disposed of. We shall assume that these CDD poles are reduced to the minimum possible and that there are no extraneous arbitrary parameters in the theory.

The cuts which are removed by this method are the cuts in the $q\bar{q}$ amplitude corresponding to decay into unbound quarks and antiquarks. There is a series of corrections [3], of order N^{-1}, which will introduce physical cuts and thus give a width to all unstable particles. We do not treat these, and thus, to use old-fashioned language, may be said to be working in the "narrow resonance" approximation.

* By naive bag or bag we mean only that confinement occurs through a boundary condition at some radius R. None of the intricacies of the MIT bag (see ref. [9]) are implied.

** We believe that the true $\tilde{G}(t)$ inserted into our algorithm would reproduce itself, and that using non-perturbative contributions in the "asymptotic" two-point function does not imply double counting of the confinement effects brought in through the cut-removal part of the algorithm. In fact it is the (presumably non-perturbative) inverse power corrections which will allow us to eliminate the "bag radius" R.

It turns out that the moment conditions depend on whether one treats the functions as having analyticity properties in E^2 (as for a spinless particle) or in E (as will be seen to be appropriate for spin-$\frac{1}{2}$ particles). The spinless case is the one treated earlier [4, 6]. The central result of this paper is the derivation of the moment conditions and their solution for the latter case. In sect. 2 we review the basic ideas and earlier treatment, and describe in more detail the limitations discussed above. In sect. 3 we describe the algorithm appropriate for spin-$\frac{1}{2}$ particles. In sect. 4 we discuss the corresponding trajectories in leading order and briefly explore the role of the perturbative logarithmic corrections through the anomalous dimension and with a sixth order example. With a realistic value for the QCD coupling we shall see that a not unreasonable ρ-trajectory emerges.

2. Confinement algorithm

We assume we have calculated the asymptotic approximation to the two-point function of interest, $G_A(t)$. To remove the cut from $G_A(t)$ to construct $G(t)$ we write [4]

$$G(t) = G_A(t) + \frac{N(t)}{D(t)}, \tag{2.1}$$

where $D(t)$ is an entire functions, whose zeros are the poles of $G(t)$. The choice of $N(t)$ that removes the cut is

$$\begin{aligned} N(t) &= \frac{1}{\pi}\int_{t_0}^{\infty} \frac{\mathrm{Im}\,[G(t') - G_A(t')]}{t'-t} D(t')\,\mathrm{d}t' \\ &= -\frac{1}{\pi}\int_{t_0}^{\infty} \mathrm{d}t'\, D(t') \frac{\mathrm{Im}\, G_A(t')}{t'-t}. \end{aligned} \tag{2.2}$$

Thus,

$$G(t) = G_A[t] - \frac{1}{\pi}\frac{1}{D(t)}\int_{t_0}^{\infty} \mathrm{d}t' \frac{\mathrm{Im}\, G_A(t')}{t'-t} D(t'). \tag{2.3}$$

$D(t)$ is determined by the requirement that $G(t) - G_A(t)$ vanishes faster than any inverse power of t as $|t| \to \infty$, and that $G(t)$ has poles with positive residues. This leads to a set of moment conditions [4],

$$\int_{t_0}^{\infty} t'^p\, \mathrm{Im}\, G_A(t') D(t')\,\mathrm{d}t' = 0, \qquad p = 0, 1, 2, \ldots. \tag{2.4}$$

It is convenient to choose t such that the lower limit is zero, which yields the moment conditions

$$\int_{0}^{\infty} t'^p\, \mathrm{Im}\, G_A(t') D(t')\,\mathrm{d}t' = 0, \qquad p = 0, 1, 2, \ldots. \tag{2.5}$$

The solution of the moment conditions (2.5) is discussed at length elsewhere [6] There are in general infinitely many solutions involving infinitely many constants. With the elimination of all CDD singularities, the solution is unique. It is straightforward to show that Im $G_A(t) = t^\nu$ implies that

$$D_0(\nu, t) = C(2R\sqrt{t})^{-\nu} J_\nu(2R\sqrt{t}) , \tag{2.6}$$

i.e. the Bessel function with the cut removed. The zero subscript on D_0 is used to denote that only the leading asymptotic term in Im $G_A(t)$ is used. The so far arbitrary scale parameter, R, will be discussed below.

Ref. [6] contains discussion of the general problem of the uniqueness of cut removal with minimum numbers of arbitrary constants. We believe that the rational function approximation, or moment condition method, is the most efficient from the point of view of convergence; in any case the result is unique.

We see that the poles are located at the zeros of the Bessel function. When

$$\text{Im } G_A(t) = t^\nu[1 + \lambda f_1(t) + \cdots] ,$$

then eq. (2.5) can be systematically solved either by use of a Green function [7, 10] or, for $f_i(t)$ an inverse power of t, by a simple ansatz [6] which shows $D(\nu, t)$ to be a series of Bessel functions. It is interesting to observe that the zeros of J_ν are just the energy levels for a free spinless particle in a non-relativistic infinite spherical well. This observation provides us with the direction for the search for a modified confinement algorithm: since the quarks have spin-$\frac{1}{2}$ we look for a scheme that will yield, to lowest order, a solution to the moment equations that looks like the solution of the Dirac equation with an infinite spherical well. This will be discussed in detail in sect. 3.

The scale parameter R is free. With leading power t^ν alone, R is fixed by fixing the location of the first zero, and thus all "recurrences" for a given spin are predicted. Choosing R is equivalent to choosing the radius of an infinite well or bag. In view of growing evidence that confinement occurs in a potential that is asymptotically linear, this may appear to be a very restrictive type of confinement. Actually, as soon as corrections to the t^ν behavior are available, another procedure, the α-expansion [3–5] becomes available, and there R no longer appears as a parameter. Qualitatively the role of R may be understood as follows: Consider a $G_A(t)$ given by

$$G_A(t) \sim t^\nu\left(1 + \frac{\lambda}{t^n}\right) . \tag{2.7}$$

When $n \to \infty$, the dependence on λ disappears and we get the result (2.6). The choice $n = \infty$ is, however, a well-defined choice and corresponds to a free particle with a leading interaction correction λ/t^∞ which can be shown [6] to be equivalent to a Schrödinger potential $(r/R)^\infty$, i.e. the well. R does not appear in (2.7) and it represents the heuristic introduction of a bag when we have no explicit corrections to t^ν. Corrections such as (2.7) with finite n represent the assertion of a confining potential and allow us to ignore the $t^{-\infty}$ corrections which represent the well. The α-expansion is appropriate for finite n and R no longer appears when it is applied.

The leading term in Im $G_A(t)$ describes the large-t behavior and thus the short-distance behavior of the interaction [6]. Thus the lowest lying bound states should be described satisfactorily with our procedure. The higher lying states will be determined more accurately as more corrections are added in Im $G_A(t)$.

The procedure that leads to (2.6) is inadequate when fermion operators are involved. A signal for this is that ν is typically an integer in lowest order. For example with

$$O^{(1)} = O^{(2)} = \bar{\Psi}\gamma_\mu \Psi \tag{2.8}$$

the lowest order term in Im $G_A(t)$ is calculated using the fermion loop, and when the appropriate tensor factor [7] $(g_{\mu\nu} - k_\mu k_\nu/k^2)$ is removed $\nu = 1$. The zeros of J_1 *do not* describe a free fermion in a well, for which a linear combination of $J_{1/2}$ and $J_{3/2}$ is needed*. As noted before, the asymptotic form t^ν does not contain enough information. This is evident from the fact that with scalar constituents and

$$O^{(1)} = O^{(2)} = \phi^+ \overleftrightarrow{\partial}_\mu \phi \tag{2.9}$$

one again gets $\nu = 1$, even though one would not expect the spectrum of "scalar positronium" to be related to that of "spinor positronium." Effectively one must look more deeply into the structure of the two-point function. If one looks graphically at a two-point function one sees that the imaginary part really is described by a four-point function which for the choice (2.8) describes quark-antiquark scattering in the center of momentum, in a total angular momentum 1 state with each quark having energy $E = \sqrt{k^2 + m^2}$. This is why in a complete calculation (which would have to be non-perturbative) the poles in the vacuum polarization would be those found in electron-positron scattering (positronium) in a QED calculation.

The imaginary part for the 2-point function $\tilde{G}(t)$ has different analyticity properties than are incorporated into the original program. It is in general analytic in E rather than E^2 and the cut-removal procedure, eq. (2.2), must be generalized to take this into account. The more general form takes into account left-hand and

* For example, the free single particle radial Dirac equation is solved by $F(r) = J_{1/2}(kr)$ and $G(r) = -\sqrt{(E-m)/(E+m)}\, J_{3/2}(kr)$ for $j = l + \frac{1}{2}$ and $l = 0$. All physically interesting quantities are combinations of F and G. Bag bound states are determined by $F(R) + G(R) = 0$. For the case of the *interaction* of two fermions of mass m, these wave functions and conditions will be repeated but m will be replaced by $2m$ and R by $\frac{1}{2}R$. See ref. [11]. For unequal mass constituents, Moseley and Rosen further show how to generalize.

right-hand cuts in E, as is the case for a spinning particle. Thus we could write

$$G(t) = G_A(t) - \frac{1}{\pi}\frac{1}{D_L(t)}\int_{-\infty}^{-m} dE' \frac{\mathrm{Im}\, G_A(t')D_L(E')}{E'-E}$$
$$-\frac{1}{\pi}\frac{1}{D_R(t)}\int_{m}^{\infty} dE' \frac{\mathrm{Im}\, G_A(t')D_R(E')}{E'-E}, \qquad (2.10)$$

where m is the mass of the particles that appear in the vacuum polarization, i.e., the quark mass. Note that the asymptotic approximation G_A will not distinguish negative E from positive E. Instead, we are using the fact that G and hence D should make this distinction. In our algorithm this is made explicit by the use of separate boundary conditions for $+m$ and $-m$. These boundary conditions will differ for particles of different spins.

How will the solutions reflect this analyticity structure? Left- and right-hand cuts are equivalent to D-functions which have even and odd pieces in E. Each piece will obey moment conditions like (2.5). The minimum CDD solution is determined by the minimum number of constants necessary to satisfy the boundary conditions appropriate to the spin. In this way we see that the differences between the spinless and spinning cases become only a matter of detail.

3. Spin-$\frac{1}{2}$ construction

In this section we study the modifications necessary when the absorptive part of the 2-point function involves fermion-fermion scattering. We shall show that the lowest order solution gives zeros at the bound states of a fermion in a naive bag, just as the lowest order solution for the spinless particle gave zeros at the bound states of a scalar particle in a naive bag.

We wish to remove a cut in E rather than E^2 in the calculated Green function G_A. This leads, as in eq. (1.4), to the new moment conditions

$$\left(\int_{-\infty}^{m} + \int_{m}^{\infty}\right)((\mathrm{Im}\, G_A)E^p D(E)\, dE) = 0\,, \qquad p = 0, 1, \dots, \qquad (3.1)$$

or equivalently

$$\int_{m}^{\infty} \mathrm{Im}\, G_A[E^p D(\nu, E) - (-E)^p D(\nu, -E)]\, dE = 0\,, \qquad p = 0, 1, \dots. \qquad (3.2)$$

If we split D into parts even and odd in E,

$$D(\nu, E) = D_a(\boldsymbol{k}^2) + E D_b(\boldsymbol{k}^2), \qquad (3.3)$$

we find, using $k\,dk = E\,dE$

$$\int_0^{\infty} (\mathrm{Im}\, G_A) k^{2n+1} D_{a,b}(k^2)\, dk = 0\,, \qquad n = 0, 1, \dots. \qquad (3.4)$$

The functions D_a and D_b obey the same moment conditions as the functions D in the previous discussion.

Let us suppose now that $\operatorname{Im} G_A = t^\nu$, where we now make the convenient choice $t = \frac{1}{4}k^2$. In this case, the solution to (2.5) is given by

$$D_a^{(0)} = k^{-\nu}[a_0 J_\nu(kR) + a_1 k J_{\nu+1}(kR) + \cdots], \tag{3.6a}$$

$$D_b^{(0)} = k^{-\nu}[b_0 J_\nu(kR) + b_1 k J_{\nu+1}(kR) + \cdots]. \tag{3.6b}$$

The infinite number of constants in eqs. (3.6) is equivalent to the CDD ambiguity. The assumption of minimal asymptotics for the D-functions, which corresponds to the usual dynamical constraint that hadrons are dynamically determined, reduces these constants to a minimum number. In fact, as we show below, the minimum condition is b_0, a_0, and a_1 non-zero. Choosing $b_0 = 1$, which is an arbitrary normalization for $D_0 = D_a^{(0)} + ED_b^{(0)}$, we have

$$D_0 = k^{-\nu}[(a_0 + E)J_\nu(kR) + a_1 k J_{\nu+1}(kR)]. \tag{3.7}$$

The Green function itself is now determined by the requirement that $G \to k^{2\nu}$ + exponentially decreasing terms, namely

$$\begin{aligned} G(E, \nu) &= k^{2\nu} \frac{(a_0 + E)J_{-\nu}(kR) - a_1 k J_{-\nu-1}(kR)}{(a_0 + E)J_\nu(kR) + a_1 k J_{\nu+1}(kR)} \\ &= \frac{D_0(-\nu, E)}{D_0(\nu, E)}. \end{aligned} \tag{3.8}$$

We can call on our experience with solutions of the Dirac equation to generalize (3.8) and define two new Green functions,

$$G^\pm(\nu, E) = \frac{D^\mp(-\nu, E)}{D^\pm(\nu, E)}, \tag{3.9}$$

corresponding to $j = \nu = l + \frac{1}{2}$ and $j = \nu - 1 = l - \frac{1}{2}$ for G^+ and G^-, respectively. D^+ is the combination $D_a + ED_b$ defined above; D^- is a different combination which is given by eq. (3.8) and the Dirac "threshold" conditions below.

Our conditions for $G^\pm$ will be

$$G^\mp \to \begin{cases} k^0, & E \to m, \\ k^{\mp 2}, & E \to -m. \end{cases} \tag{3.10}$$

The condition at $E \to +m$ gives us nothing new; that at $E \to -m$ determines $a_0 = m$. The constant a_1 remains as yet undetermined.

We can find a_1 by using a MacDowell-type symmetry that a free Dirac particle must obey,

$$-k^2 G^+(\nu, -E) = G^-(\nu + 1, E). \tag{3.11}$$

This equation refers to states of the same j but of opposite parity $(-1)^l$. When applied to eqs. (3.8), the symmetry gives immediately $a_1^2 = 1$. The additional requirement that both solutions $E^{\pm}$ for the bound states at $D^{\pm} = 0$ approach m from above chooses the root $a_1 = -1$.

In summary, the leading order D-functions are

$$D_0^+(\nu, E) = k^{-\nu}[(E+m)J_\nu(kR) - kJ_{\nu+1}(kR)]\,, \tag{3.12a}$$

$$D_0^-(\nu, E) = k^{-\nu}[(E+m)J_\nu(kR) + kJ_{\nu-1}(kR)]\,. \tag{3.12b}$$

Higher order corrections of the form $G_A \to t^\nu(1 + gt^{-\Delta} + \cdots)$ can now be handled analogously to the discussion in ref. [6]. In addition to the moment conditions, the $D^{\pm}$ must give $G^{\pm}$ which satisfy the conditions (3.10).

4. Perturbative corrections and numerical results

In this section we discuss the role of logarithmic corrections; these corrections arise naturally in perturbative calculations of the 2-point functions. As an example which illustrates the numerical effect of these corrections we consider up to the sixth order in the 2-point function constructed with $O^{(1)} = O^{(2)} = \bar\psi\gamma_\mu\psi$. This quantity, which is of direct interest for $\sigma(\text{hadrons})/\sigma(\ell^+\ell^-)$ measured in e^+e^- annihilation, has been recently calculated [12]. The results of this calculation are inserted into the confining algorithm described in sect. 3 and thus provide a perturbative correction to the bag for fixed radius R. We find that the correction is numerically small and moreover provides only a constant shift in the position of the high-lying poles. This means the trajectory can never become linear as a result of the use of a single logarithmic correction.

The single logarithm can be handled in two equivalent ways. Let us suppose that we are dealing with a function of the type

$$\operatorname{Im} f = At^\nu[1 + \lambda \ln(4R^2t)]\,. \tag{4.1}$$

This can be regarded as a standard power correction, as in sect. 2, by writing

$$\operatorname{Im} f = At^\nu\left[1 + \lambda \frac{\mathrm{d}}{\mathrm{d}\varepsilon}(4R^2t)^\varepsilon\bigg|_{\varepsilon=0}\right]. \tag{4.2}$$

The derivative with respect to ε can be taken at the end. Alternatively, we have

$$(4R^2t)^\nu[1 + \lambda \ln(4R^2t)] = (4R^2t)^{\nu+\lambda}[1 + \mathrm{O}(\lambda^2)]\,. \tag{4.3}$$

This is the zeroth order form with a shifted value of ν. Both eqs. (4.2) and (4.3) are equivalent because they lead to the same values of the bound-state poles. We shall discuss a concrete example of such a correction below. For now we assume eq. (4.1) holds and discuss the corresponding zeros.

There are various ways to locate the zeros of the $D^{\pm}$ function, corresponding to $\nu = l + \frac{1}{2} = j$. For us it is most convenient simply to plot the zeros of $D_0^+(\nu)$ for a sequence of discrete values of ν, then to interpolate to the zeros of $D_0^+(\nu+\lambda)$. The relevant zeros for D^+ in our example will be at $j=\frac{1}{2}+\lambda$. The zeros of $D_0^+(\nu)$ are given by the zeros of

$$d_0^+(\nu)=(E+m)J_\nu(2R\sqrt{t})-2\sqrt{t}J_{\nu+1}(2R\sqrt{t})\,, \tag{4.4}$$

or of

$$\begin{aligned}\bar{d}_0^+(\nu)&=d_0^+(\nu)R\\&=[\sqrt{z^2+\mu^2}+\mu]J_\nu(z)-zJ_{\nu+1}(z)\,.\end{aligned} \tag{4.5}$$

In eq. (4.5), we have defined the dimensionless variables

$$z\equiv 2R\sqrt{t}=\sqrt{E^2-m^2}R\,, \tag{4.6}$$

$$\mu\equiv mR\,, \tag{4.7}$$

which are parameters in the problem.

We plot $\bar{d}_0^+$ as a function of z for various ν. The zeros of this function can then be located and plotted for ν versus z; this last plot represents a Regge trajectory. Fig. 1 shows a sample plot of $\bar{d}_0^+$ of ν for the case $\nu=0$. Fig. 2 gives the trajectories, given the zeros for $\nu=0$, 1, and 2 read from graphs like fig. 1.

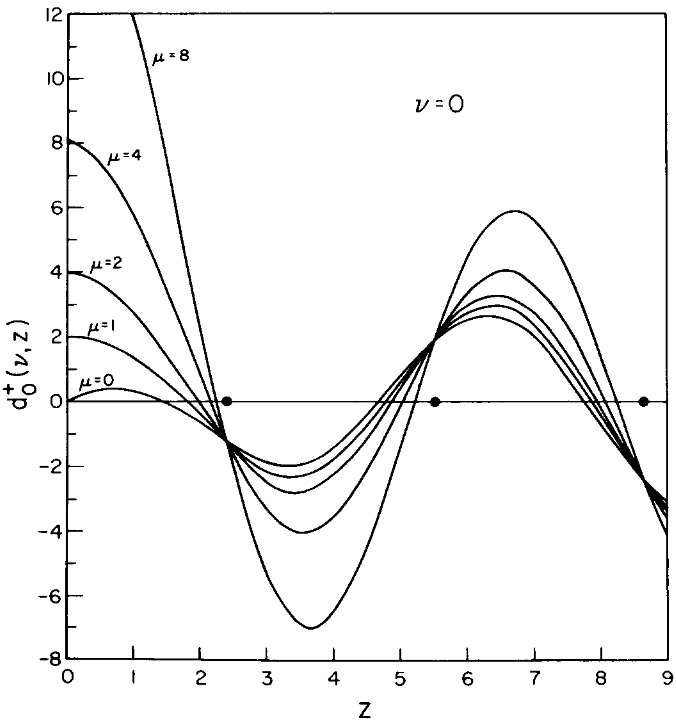

Fig. 1. The function $d_0^+(\nu,z)$, eq. (4.4), with $\nu=0$, as a function of $z\equiv\sqrt{E^2-m^2}R$ for various values of $\mu\equiv mR$. The zeros of $J_0(z)$ are marked by a full dot and represent the zeros of $d_0^+(0,z)$ in the limit $\mu\to\infty$.

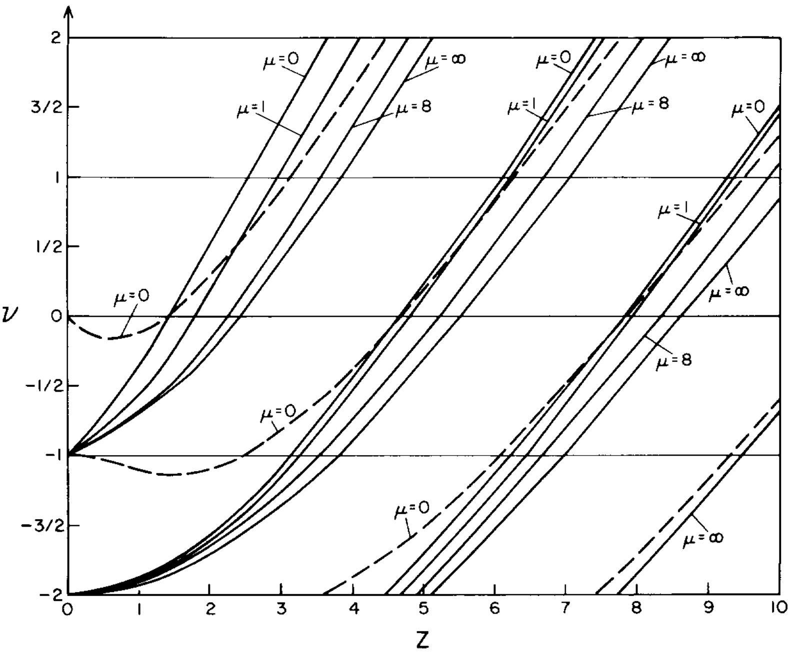

Fig. 2. The ν-trajectories, solid lines, for $d_0^+(\nu, z)$, $j=\nu=l+\frac{1}{2}$, $z=kR$ and $\mu \equiv mR = 0, 1, 8, \infty$. The zeros of $d_0^-(\nu, z)$, dashed line, $j=\nu-1=l-\frac{1}{2}$ for $\mu=0$. When $\mu \to \infty$, $d_0^+(\nu, z) = d_0^-(\nu, z)$, i.e. there is no "spin-orbit" splitting. Note the sign of the spin-orbit term corresponds to scalar confinement.

The trajectories become linear in z for large z. This can be seen by (self-consistently) assuming zeros at $z=O(\nu)$. We define α by $z=\nu \operatorname{sech}\alpha$ (i.e. $z \leqslant \nu$; we get only the leading trajectory this way) and can use Debye asymptotic expansions for J_ν ($\nu \operatorname{sech}\alpha$) and J'_ν ($\nu \operatorname{sech}\alpha$) in the alternative form

$$\bar{d}_0^+ = (\sqrt{z^2+\mu^2}-\mu)J_\nu - z\left(\frac{\nu}{z}J_\nu - J'_\nu\right). \tag{4.8}$$

For $\mu=0$, the equation $\bar{d}_0^+$ has solution $z=\nu$. For $\mu=\infty$, the equation $\bar{d}_0^+$ is the solution of $J_{\nu+1}(z)=0$, i.e. $z=\nu+1$. Comparison with fig. 2 shows that these asymptotic forms are already well approximated at low values of ν. Of course a plot linear in $z=2R\sqrt{t}$ versus ν will be of the form $\nu \sim \sqrt{t}$, characteristic of the well, compared to the conventional Regge trajectory linear in t.

For a specific numerical example we could set $\nu=\frac{1}{2}$ and fit m and R to the ρ and ρ' states. We find the reasonable values $R=0.72$ fm and $m=370$ MeV. For comparison with the one particle bag parameters it should be born in mind that m corresponds to twice the quark mass and R to half the bag radius [11]. We can predict the location of the next recurrence at $M=2.45$ GeV but since in zeroth order our confinement is in a square well, this state is very likely too high.

For a specific example of logarithmic corrections and their effects, we turn to the 2-point function

$$\langle j_\mu(x) j_\nu(0)\rangle = \langle \bar{\Psi}(x)\gamma_\mu \bar{\Psi}(x)\Psi(0)\gamma_\nu\Psi(0)\rangle\,. \tag{4.9}$$

This function has recently been calculated [12] to sixth order in the coupling for QCD in SU(N) of color and with an arbitrary number N_f of flavors. The result for large t is, to O(g^6),

$$\operatorname{Im} f \sim (tR^2)\{1+\zeta \ln (tR^2)\}\,, \tag{4.10}$$

where

$$\zeta = 3b_1\bar{\alpha}^2\,, \tag{4.11a}$$

$$\bar{\alpha} \equiv \alpha N = \frac{g^2}{4\pi} N\,, \tag{4.11b}$$

$$b_1 = -\frac{1}{96\pi^2}\left(1-\frac{1}{N^2}\right)\left(11-2\frac{N_f}{N}\right). \tag{4.11c}$$

Although the full answer for (4.9) in ref. [12] is renormalization method dependent, this dependence can be absorbed into the irrelevant (for us) constant in front of (4.10). There is also a single power of α in this overall constant.

The numerical effect of (4.10) is very small. For $N=3$, $N_f=6$ and, say, $\alpha=0.5$, $\zeta=-0.04$. Such a shift from $\nu=\frac{1}{2}$ is virtually invisible on fig. 2. In particular the *spacing* of the trajectories for some given μ is a second order effect in ζ.

For a more general 2-point function the O($\bar{\alpha}$) term will have a logarithm, due to the anomalous dimension γ_n. Namely, we expect [13] in general to O(g^4)

$$\operatorname{Im} f \sim t^n \left\{1+\frac{\bar{\alpha}}{2\pi}\gamma_n \ln (tR^2)\right\}, \tag{4.12}$$

$$\gamma_n = \frac{1}{2} - \frac{1}{n(n+1)} + 2\sum_{l=1}^{\infty} \frac{(n-1)}{(l+n)(l+1)}\,. \tag{4.13}$$

This shifts the power n in a way that depends on n. For $n=1$, the case considered above, $\gamma_n=0$, in accordance with the result (4.10).

Fig. 3 shows the result of inclusion of the anomalous dimension, again using $\alpha=0.5$ ($\bar{\alpha}/2\pi=0.24$). All other parameters are fixed from the zeroth order fit to ρ and ρ', namely $m=0.37$ GeV and $R=0.72$ fm. We also include on this plot the particles* lying on the trajectory. The change is in the right direction. Note that

* Ref. [13] makes precise the operators which correspond to the anomalous dimensions, eq. (4.13), namely the twist-two traceless (or symmetric tensor) operators. For these operators the asymptotic behavior of Im f, eq. (4.12), is a continuous function of ν and describes the so-called "vector" trajectory. The A_2 shown in fig. 3 is assumed to correspond to this set of operators, as does the ρ. There will presumably be other trajectories corresponding to other sets of operators.

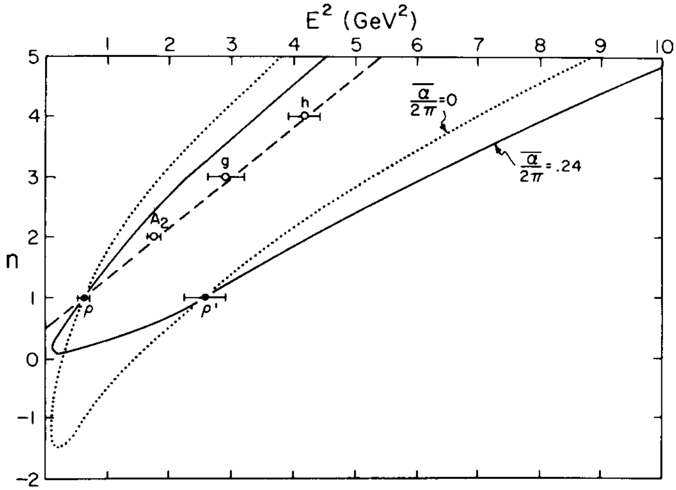

Fig. 3. The ρ, ρ' Regge trajectories to order $\bar{\alpha}$. The ρ and ρ' (input) are marked by full dots. The $(\bar{\alpha}/2\pi)=0$ trajectory, fig. 2, with $n=\nu+\frac{1}{2}$, $\mu=mR=1.34$, $m=0.37$ GeV, $R=0.7$ fm is shown by the dotted line. The solid line represents the trajectory with anomalous dimension correction, eqs. (4.12), (4.13), with $(\bar{\alpha}/2\pi)=0.24$. The dashed line shows the function $n(E)=0.5+0.83E^2$ for reference.

since the asymptotic form of the trajectory with anomalous dimension correction is $\nu\sim E$, the plotted trajectory will eventually turn over and cross the linear trajectory. However, this may happen at a point where the perturbative result is no longer valid, since eq. (4.13) can be rewritten as

$$\gamma_n=-0.346+2\ln n-2\sum_{k=2}^{\infty}\frac{A_k}{n(n+1)-(n+k-1)}, \tag{4.14}$$

where the A_k are a set of tabulated numbers. The $\ln n$ term implies that for sufficiently large n, $(\bar{\alpha}/2\pi)\gamma_n$ is not small.

We could have taken an alternative tack, by fitting $\bar{\alpha}/2\pi$ to a reasonable slope near $n=1$. This would give $\bar{\alpha}/2\pi\approx0.8$, which corresponds to a QCD coupling constant high by a factor of roughly two. We do not take this approach because we expect that when power corrections come into play the ρ trajectory will straighten out and the same phenomenology will ask for a much smaller $\bar{\alpha}/2\pi$.

Finally, we could also have done the same phenomenology for the J/ψ and Υ Regge trajectories. The results are qualitatively very similar, the only change being different values of m and R in eq. (4.4). We reserve extended confrontation with the meson spectrum for the next stage, where we shall include the true confining force, which corresponds to corrections of order $1/t^2$.

5. Conclusions

The procedure we have discussed here is in some sense analogous to the development of perturbative quantum field theory. By this we mean the following: In perturbation theory we begin with free field quantities, such as propagators. Interactions are then introduced; they are described in terms of the free field propagators and some additional quantities, the interactions. In our case we start from a given leading asymptotic behavior, t^{ν}, and develop a two-point function which corresponds to a free field in a confining well. The D-function D_0 which describes this behavior is analogous to the free propagator in quantum field theory. The role of interactions, which are manifested in corrections to the leading asymptotic behavior, then give a corrected two-point function; the corresponding correction to the D-function can be expressed in terms of D_0 and the correction to the asymptotic behavior. Just as interactions in quantum field theory lead, through renormalization, to a corrected set of parameters for the "free" field quantities, so too do the corrections to the leading asymptotic behavior lead to confining behavior which is closer to reality than the original infinite well confinement imposed in computation of D.

Free field propagators vary according to the spin of the propagating particle. We similarly have found that D_0 must be chosen through appropriate analyticity behavior, according to the spin of the fundamental constituents of the operators making up the two-point functions. This behavior will then be naturally carried over into the higher order calculations. We have in this paper shown how additional information about the spin of the particle assumed to be confined manifests itself through analyticity properties, and in this way we have developed a connection between asymptotic behavior of a two-point function and the location of the bound states appropriate for spin-$\frac{1}{2}$ constituents. This carries the initial work to a point where a realistic phenomenology of the spectrum of QCD can begin.

We have looked at one piece of this phenomenology in this note. Corrections to the behavior t^{ν} are of two types: non-perturbative power corrections and perturbative logarithmic corrections. The logarithmic corrections are short-range and will not by themselves be responsible for confinement. Nevertheless it is interesting to see if they have a substantial numerical effect on the location of the poles. Our conclusion is that they do not beyond second order; we have arrived at this conclusion by studying a specific two-point function for spin-$\frac{1}{2}$ particles calculated to sixth order. The effects of the anomalous dimension for arbitrary angular momentum is substantial, although linearization of the trajectory will only come from power-type corrections. Contributions from power corrections represent a major area to be explored in the future. These come from fermion mass terms, i.e. from chiral symmetry breaking, and from other effects of the non-perturbative vacuum.

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